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MODELING THE DELAY IN MIXING TO THE MOLECULAR LEVEL IN CALCULATING THE THERMAL RADIATION OF TURBULENT FLOWS

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An appropriate method is proposed for taking account of the delay in mixing to the molecular level in turbulent flows of jet type; the method is intended for the calculation of the thermal radiation.

In calculating the physical and chemical processes occurring in free jets and wakes with the decay of tangential disturbances, with a nonlinear dependence on the thermodynamic parameters, there arises the problem of taking correct account of the problem of turbulent mixing, which influences the structure of the concentration and temperature fields [1-3]. One of the most important manifestations of the flow turbulence is the delay in mixing to the molecular level, i.e., the retention of the individual characteristics (temperature, chemical composition, etc.) of the volume of material penetrating into the turbulized region for a finite interval of time. In [3], it was shown that the problem of taking turbulence into account in calculating the chemical-reaction rates in peripheral regions of turbulent jets may be successfully solved by direct modeling of the mixing delay.

Such modeling may also be required in calculating the thermal radiation of a high-temperature jet issuing into a medium at a lower temperature and also in developing optical methods of turbulent-flow diagnostics. This is associated with the fact that finite volumes of unmixed hot gas penetrate from the potential core of the flow into the peripheral regions of the jet, with a certain probability. Neglecting this phenomenon may significantly affect estimates of the transverse dimensions of the jet with respect to the thermal radiation or the calculation of the brightness-temperature fields. Quantitative estimates are required for more definite assertions regarding the influence of the mixing delay on the thermal radiation. A corresponding method for numerical investigations will be constructed, following [3], on the basis of a two-parameter model of the turbulent viscosity.

Numerous variations of the two-parameter model of turbulent viscosity (i.e., models in which the turbulent viscosity is determined by two characteristic parameters of the turbulence field) are based on the kinetic-energy balance equation of the pulsations, to which is added the differential equation determining the scale of the pulsations. The basis of the two-parameter model, its fundamental equations, and also some examples of its use may be found in [4], for example. Critical analysis of the principal modifications of the model based on a comparison of experimental and theoretical data may be found in [5].

So as to be specific, isobaric axisymmetric jets will be considered below. The so-called $\langle k\varepsilon 2\rangle$ model of turbulent viscosity will be used for their calculation [5]. One of the advantages of this model is that, in selecting the empirical coefficients, much attention is given to the description of sections of the flow remote from the axis, which is important in calculating the radiation in peripheral regions of the jet. Within the framework of the $\langle k\varepsilon 2\rangle$ model, the dissipation of the kinetic energy of the pulsations ε is used as the function determining the characteristic scale of the pulsations. The turbulent viscosity is

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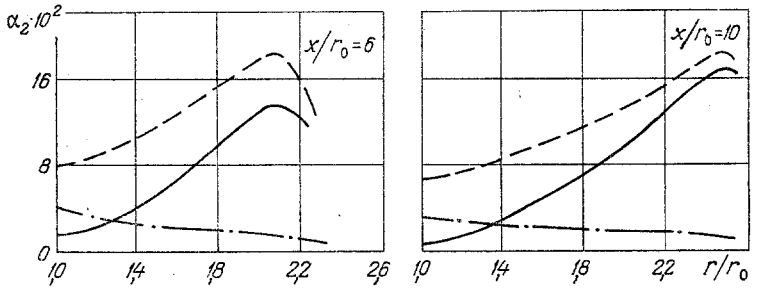

Fig. 1. Transverse distribution of the degree of nonmixing of the gas of the external space α_2 for a submerged jet of water vapor in air (t_0 = 600°K). In the calculations Eqs. (12)-(15) are used (continuous curves), as well as Eq. (11) (dash-dot curves), and the method of [3] (dashed curves).

specified by the following system of equations (axial symmetry, boundary-layer approximation)

$$\rho u\frac{\partial k}{\partial x}+\rho v\frac{\partial k}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial r}\right)+\rho\nu_t\left(\frac{\partial u}{\partial r}\right)^2-\rho\varepsilon, \tag{1}$$

$$\rho u\frac{\partial \varepsilon}{\partial x}+\rho v\frac{\partial \varepsilon}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial r}\right)+C_{\varepsilon 1}\rho\nu_t\left(\frac{\partial u}{\partial r}\right)^2-C_{\varepsilon 2}\frac{\rho\varepsilon^2}{k}, \tag{2}$$

$$\nu_t=C_\mu k^2/\varepsilon. \tag{3}$$

The mean values of the velocity, temperature, and mass concentration are calculated on the basis of the standard system of equations for an isobaric axisymmetric turbulent flow [6]

$$\frac{\partial}{\partial x}(\rho u)+\frac{1}{r}\frac{\partial}{\partial r}(\rho v r)=0, \tag{4}$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho\nu_t\frac{\partial u}{\partial r}\right), \tag{5}$$

$$\rho u\frac{\partial H}{\partial x}+\rho v\frac{\partial H}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho\frac{\nu_t}{\sigma_H}\frac{\partial H}{\partial r}\right)+\left(1-\frac{1}{\sigma_H}\right)\frac{1}{r}\frac{\partial}{\partial r}\left[r\rho\nu_t\frac{\partial}{\partial r}\left(k+\frac{u^2}{2}\right)\right], \tag{6}$$

$$\rho u\frac{\partial C_i}{\partial x}+\rho v\frac{\partial C_i}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho\frac{\nu_t}{\sigma_C}\frac{\partial C_i}{\partial r}\right)\quad (i=1,\ 2,\ \ldots,\ N). \tag{7}$$

The relation between the stagnation enthalpy H and the other quantities is determined in the form $dH=\left(\sum_i c_{p,i}C_i\right)dT+\sum_i h_i dC_i+d(u^2/2)+dk$. The empirical constants σ_H and σ_C (Prandtl and Schmidt numbers) are assumed to be 0.7.

The system in Eqs. (1)-(7) is closed by the equation of state of an ideal gas. The boundary conditions for Eqs. (1)-(7) are determined: a) by the known values of the flow parameters in the external flow; b) by the axial symmetry of the flow.

The properties of the two-potential model allow a semiempirical equation to be constucted for the mean-square deviation of the concentration g [6]. Within the framework of the <kε2> model, this equation takes the form

$$\rho u\frac{\partial g}{\partial x}+\rho v\frac{\partial g}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho\frac{\nu_t}{\sigma_g}\frac{\partial g}{\partial r}\right)+C_{g1}\rho\nu_t\left(\frac{\partial C}{\partial r}\right)^2-C_{g2}\rho\frac{\varepsilon}{k}g. \tag{8}$$

The quantity C, for which the mean-square deviation g is calculated, is understood to be the mass concentration of the mixture issuing from the nozzle. (In the potential core of the flow, it is equal to unity and in the external medium to zero.) In accordance with the recommendations of [6], based on a comparison with experimental data, the empirical constants

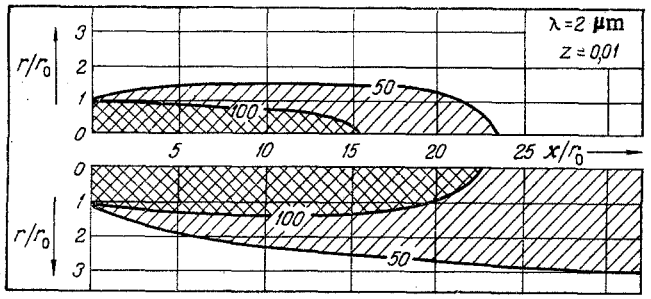

Fig. 2. Isotherms of the level of the brightness temperatures above the background for calculations from the means T and C (at top) and taking account of the mixing delay by means of a model (at bottom).

σ_g, C_{g1}, and C_{g2} are taken to be equal to σ_ε, $C_{\varepsilon 1}$, and $C_{\varepsilon 2}$, respectively. The last term on the right-hand side of Eq. (8) denotes the dissipation of g on account of molecular mixing, and may be written in the form g/τ, where τ is the characteristic time of such mixing.

In [3], it was shown that effects associated with delay in mixing to the molecular level may be approximately taken into account using information on C and g. The basis for this procedure is the assumption that the set of different states of the flow gas realized at a given point of the turbulent medium may be modeled in the form of a superposition of two components: the unmixed gas of the external space and homogeneous gas mixture. In calculating chemical reactions in the peripheral regions of the jet, this approach is justified, since the efficiency of mixing of the oxidant from the external space has a decisive influence in this case. In calculating the radiation for these regions of the jet, there arises the additional problem of taking account of the penetration of unmixed gas from the potential core of the flow, since even small "impregnations" of unmixed hot gas may have a significant influence on the radiation in the case of high temperature in the nozzle cross section. To take this into account, while remaining at the same level of modeling, it is assumed that the set of realizations of an elementary volume of the turbulent medium may be represented in the form of a superposition of three noninteracting components: the gas of the external space and the working gas of the jet, which completely retain their individual properties, and also a homogeneous mixture of the first two components. The relative time intervals in the course of which the working gas of the jet and the gas of the external space, with no internal modifications, are working at the given point are denoted by α_1 and α_2, respectively, while the mass concentration of the gas issuing from the nozzle, mixing to the molecular level, is denoted by C. Then the following relations may be written

$$C = (1 - \alpha_1 - \alpha_2)\,\tilde{C} + \alpha_1, \tag{9}$$

$$g = (1 - \alpha_1 - \alpha_2)(C - \tilde{C})^2 + \alpha_1 (C - 1)^2 + \alpha_2 C^2. \tag{10}$$

For closure of the given system of equations, additional relations between α_1, and $\tilde{C}$ must be used. As a first step, it is natural to assume that the ratio α_1/α_2 coincides with the ratio of the corresponding mass concentrations

$$\alpha_1/\alpha_2 = C/(1 - C). \tag{11}$$

It is obvious that for regions of the flow remote from the axis this assumption leads to overestimates for α_1. This is also confirmed in that the value of α_2 obtained using Eq. (11) does not agree with the corresponding asymptotic values of the "degree of unmixedness" as the distance from the potential core increases [3] (Fig. 1). The basic reason for this discrepancy is that differences between α_1 and α_2 due solely to turbulent diffusion are taken into account in Eq. (11). With increasing progress into the limits of the mixing zone, the values of α_1 and α_2 for the corresponding gas volumes are also changed as a result of molecular mixing. This may be taken into account as follows. Suppose that Eq. (11) is replaced by the relation

$$\alpha_1/\alpha_2 = \alpha_1^*/\alpha_2^*, \tag{12}$$

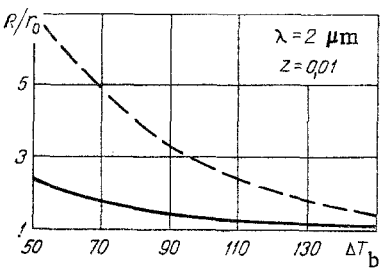

Fig. 3. Dependence of the isotherms radius on the level of the brightness temperature above the background for radiation directed in the anterior hemisphere. Calculations from the means T and C (continuous curve) and taking account of mixing delay by means of a model (dashed curve).

where α_1^* and α_2^* are quantities which change because of the turbulent diffusion analogously to the quantities C and 1 − C but at the same time also change on account of mixing to the molecular level with a characteristic time τ^*, i.e., α_1^* and α_2^* satisfy the equations

$$\rho u \frac{\partial \alpha_1^*}{\partial x} + \rho v \frac{\partial \alpha_1^*}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(\rho \frac{\nu_t}{\sigma_C} \frac{\partial \alpha_1^*}{\partial r} \right) - \rho \frac{\alpha_1^*}{\tau^*}, \tag{13}$$

$$\rho u \frac{\partial \alpha_2^*}{\partial x} + \rho v \frac{\partial \alpha_2^*}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(\rho \frac{\nu_t}{\sigma_C} \frac{\partial \alpha_2^*}{\partial r} \right) - \rho \frac{\alpha_2^*}{\tau^*}, \tag{14}$$

while the initial and final conditions for α_1^* and α_2^* are analogous to the initial and boundary conditions for C and 1 − C. Assuming that τ^* coincides with the characteristic time of dissipation of the desired quantities α_1 and α_2, it is expressed in terms of τ, the characteristic dissipation time of g. This involves writing the right- and left-hand sides of Eqs. (9) and (10) due to mixing to the molecular level. The value of the mean concentration C remains constant, since it does not depend on the degree of mixing within the limits of the elementary volume

$$0 = (1 - \alpha_1 - \alpha_2)\tilde{C} + \left(\frac{\alpha_1}{\tau^*} + \frac{\alpha_2}{\tau^*} \right) \tilde{C} - \frac{\alpha_1}{\tau^*},$$

$$-\frac{g}{\tau} = -2(1 - \alpha_1 - \alpha_2)(C - \tilde{C})\dot{\tilde{C}} + \left(\frac{\alpha_1}{\tau^*} + \frac{\alpha_2}{\tau^*} \right)(C - \tilde{C})^2$$

$$-\frac{\alpha_1}{\tau^*}(C - 1)^2 - \frac{\alpha_2}{\tau^*} C^2.$$

Excluding $\dot{\tilde{C}}$, the rate of change in $\tilde{C}$ associated with molecular mixing, from these relations, it is found that

$$\frac{\tau^*}{\tau} = 1 + \frac{(C - \tilde{C})^2}{g}. \tag{15}$$

Thus, Eqs. (9), (10), and (12-(15) constitute a closed system of equations allowing α_1, α_2, and C to be calculated. As is evident from Fig. 1, the asymptotic behavior of α_2 agrees considerably better with the results of [3] when this method of closure is used. This permits the hope that the method here proposed, based on the use of Eqs. (9), (10), and (12)-(15), provides a sufficiently good description of the features of mixing in turbulent flows that are of interest.

In using the method developed here to investigate the influence of the mixing delay on the thermal radiation of turbulent jets, attention is limited to those media where: 1) local thermodynamic equilibrium is realized; 2) the scale of the inhomogeneities due to mixing delay is considerably less than the distances at which the radiation field undergoes significant changes. In this case, the intensity of the monochromatic radiation I is determined by the equation

$$\frac{dI}{ds} = \varkappa B - \varkappa I. \tag{16}$$

Taking mixing delay into account reduces to averaging the absorption coefficient $\varkappa$ and the source function $\varkappa B$ within the framework of the given model for an elementary volume of the turbulent medium.

It is inexpedient to consider real gases as the radiating medium in the first stages. In addition to the unavoidable technical difficulties, this is because the selective character of the radiation and the complex temperature dependence of the absorption cross section complicate the analysis of effects associated directly with mixing delay. Therefore, the subsequent investigations will be based on a model of the radiating medium. It is as-

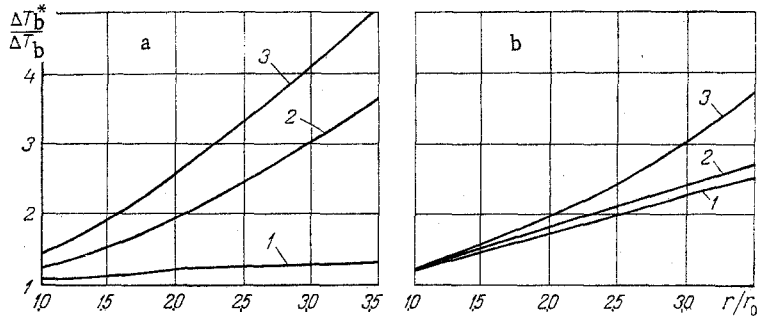

Fig. 4. Dependence of the contribution of unmixed hot gas to the radiation on the wavelength (a) and optical thickness (b): a) z = 0.01; λ = 4 μm (1), 2 (2), 1 (3); b) λ = 2 μm; z = 0.1 (1), 0.05 (2), 0.01 (3).

sumed that from a gas-dynamic viewpoint the radiating components form a passive impurity of temperature equal to that of the surrounding gas. The dependence of the absorption cross section on the wavelength λ and temperature will be neglected.

For the case when there are no radiating components outside the jet, the source function, averaged by the given method, takes the form

$$\langle \varkappa B \rangle = \alpha_1 \varkappa (T_0, C_0) B (T_0) + (1 - \alpha_1 - \alpha_2) \varkappa (\tilde{T}, \tilde{C}, C_0) B (\tilde{T}). \tag{17}$$

Since, according to numerical estimates, α_1 and α_2 are much less than unity, as a rule, while $\tilde{T}$ and $\tilde{C}$ differ only slightly from the corresponding mean mass values T and C, Eq. (17) may be approximately written in the form

$$\langle \varkappa B \rangle \approx \varkappa (T, C C_0) B (T) \left[1 + \alpha_1 \frac{\varkappa (T_0, C_0) B (T_0)}{\varkappa (T, C C_0) B (T)} \right]. \tag{18}$$

For the given model of the radiating medium, the dependence of the absorption coefficient on the thermodynamic parameters is smoother than the analogous dependence for the Planck function, so that it may be assumed, for preliminary estimates, that $\langle \varkappa \rangle = \varkappa(T, C, C_0)$. Thus, the mixing delay may be approximately taken into account by multiplying the source function calculated from the means T and C by an effective factor: $[1 + \alpha_1 \varkappa(T_0, C_0)B(T_0)/(\varkappa(T, C C_0) \cdot B(T))]$.

Such estimates confirm the fairly obvious supposition that the influence of mixing delay on the radiation should increase with decrease in wavelength. The corresponding behavior of the optical thickness is not so obvious. However, the role of mixing delay may evidently be expected to decrease with increase in optical thickness, since radiation from small "impregnations" of unmixed hot gas will absorb more effectively than the colder components in this case.

Numerical investigations of the influence of mixing delay on the thermal radiation of jets are performed in the following sequence. By solving Eqs. (1)-(8), the mean mass characteristics of the jet are determined. The corresponding calculation algorithm is constructed on the basis of the method of [7], which was specially developed for numerical investigations of turbulent flow in the boundary-layer approximation. Simultaneously, on the basis of Eqs. (9), (10), and (12)-(15), the parameters of the model for approximately taking account of the mixing delay are calculated; these are used for averaging the absorption coefficient and the source function within the limits of an elementary volume of the turbulent medium. Equation (16) is integrated both for $\varkappa$ and $\varkappa B$ calculated from the means T and C, and for the quantities $\langle \varkappa \rangle$ and $\langle \varkappa B \rangle$ averaged within the framework of the given model.

Individual results of the numerical investigations are shown in Figs. 2-4. They are obtained for a submerged jet with an initial temperature T_0 = 800°K, an initial velocity u_0 = 120 m/sec, and an initial-mixture molecular weight μ_0 = 20 g/mole. The temperature of the surrounding air is assumed to be 300°K. The wavelength of the radiation varies from 1 to 4 μm. It may readily be shown that, for an isobaric jet, the optical thickness may be characterized by a parameter that is constant for any specific flow: $z \sim p r_0 C_0$. The dependence of the influence of mixing delay on the optical thickness will be analyzed by varying z. As an example, note that, if the radiation from the jet is determined by soot particles,

then $z = 0.01$ (with $\lambda = 2$ μm, $p = 1$ atm, and $r_0 = 0.1$ m) corresponds to approximately 0.01% (by mass) soot in the initial mixture. For $\lambda = 4$ μm, with the same values of z, r_0 and p, the corresponding mass fraction of soot should be approximately three times larger.

Regions for which the level of the brightness temperature above the background (blackbody background is assumed) is no less than 50 and 100°K, respectively, at $\lambda = 2$ μm are shown in Fig. 2. The given results correspond to scanning of the jet "from the side"; the beams for which Eq. (16) is integrated and the jet axis are found to be perpendicular on reducing to a single plane. The upper half of Fig. 2 corresponds to calculation on the basis of the mean concentration and temperature values, and the lower half to calculation taking the mixing delay into account by means of a model. As may be seen, neglecting the presence of unmixed gas introduces significant errors in calculating the brightness-temperature field of a high-temperature turbulent jet.

Results characterizing radiation directed into the anterior hemisphere parallel to the jet axis are shown in Fig. 3; the radii of isotherms of the level of the brightness temperature above the background in the form of circles are compared. Again it may be noted that neglecting the mixing delay leads to significantly underestimated results in calculating the brightness-temperature field. The dependence of the contribution of unmixed hot gas to the radiation on the wavelength and the optical thickness is shown in Fig. 4. The corresponding quantitative characteristic employed is the ratio of the brightness-temperature levels above background, calculated by two methods, as before. As is evident, numerical methods confirm the qualitative conclusions that the influence of unmixed hot gas on the radiation should increase with decrease in the wavelength of the radiation and the optical thickness.

NOTATION

x, r, cylindrical coordinates; u, v, longitudinal and transverse mean velocities; ρ, density; p, pressure; k, kinetic energy of the pulsations; ε, dissipation of the kinetic energy of the pulsations as heat; ν_t, kinetic turbulent viscosity; H, stagnation enthalpy; T, temperature; C_i, mass concentration of component i; $c_{p,i}$, specific heat at constant pressure; h_i, heat of formation of component i (with a minus sign); N, total number of chemical components in mixture; C, mass concentration of gas mixture issuing from nozzle; g, mean square deviation corresponding to C; α_1, degree of unmixedness of the working gas of the jet; α_2, degree of unmixedness of the gas of the external space; $\tilde{T}$, $\tilde{C}$, temperature and concentration of mixture issuing from nozzle and undergoing mixing to the molecular level; $\dot{\tilde{C}}$, rate of change of $\tilde{C}$ on account of molecular mixing; τ, characteristic dissipation time of g; α_1^*, α_1^*, model quantities for establishing the relation between α_1 and α_2; T_0, C_0, r_0, temperature and mass concentration of radiating components and jet radius in initial cross section; I, intensity of radiation; λ, wavelength; z, optical-thickness parameter; ϰ, absorption coefficient; B, Planck function; s, distance along beam; C_μ, σ_k, σ_ε, σ_H, σ_C, σ_g, $C_{\varepsilon 1}$, $C_{\varepsilon 2}$, C_{g1}, and C_{g2}, empirical constants; ΔT_b, level of brightness temperature above background calculated from the means T and C; ΔT_b^*, corresponding quantity calculated taking account of the mixing delay by means of a model; R, distance of the brightness-temperature isotherm from the jet axis.

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